* R B - 0 7 2 4 / 3 0 0 0 *

RB-0724

Second Year B. Sc. (Computer Science) Examination

April / May – 2010

Maths : Paper - III

(New Course)

Time : **3** Hours] [Total Marks : **105**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ S.Y. B.Sc. (COMPUTER SCIENCE)

Name of the Subject :

☛ MATHS - 3 (NEW)

☛ Subject Code No. : 0 7 2 4 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate full marks.

1 Answer the following questions : **15**

(1) Give the complementary function for $\left(D^2-4\right)y=12$.

(2) If $x=r\cos\theta,\quad y=r\sin\theta$ then find $\dfrac{\partial(x,y)}{\partial(r,\theta)}$.

(3) Evaluate $\displaystyle\int_0^2 \frac{x^2}{\sqrt{2-x}}dx$

(4) Define Gamma function.

(5) Find the value of $\left|\overline{(1/2)}\right.$.

(6) If $y=\sin^{-1}\left(\dfrac{x}{y}\right)$ then find u_x and u_y.

(7) Define extreme values of functions.

(8) Verify the function for its homogenity;

$$f(x,y,z)=\frac{4x^3+2y^2z}{x+2y+3z}$$

(9) State Taylor's theorem for function of two variables.

(10) Describe the region of integration $\displaystyle\int_0^2\int_0^4 dx\,dy$.

2 (a) If $f\left(D^2\right)$ is a polynomial in D^2 with constant coefficients and $f\left(-a^2\right) \neq 0$ then **18**

(i) $\dfrac{1}{f\left(D^2\right)}\cos ax = \dfrac{1}{f\left(-a^2\right)}\cos ax$

(ii) $\dfrac{1}{f\left(D^2\right)}\sin ax = \dfrac{1}{f\left(-a^2\right)}\sin ax$.

(b) Solve : $\left(D^2 - 3D + 2\right)y = 2x^2 - 5x + 3$.

(c) Solve : $\left(D^2 - 2D + 1\right)y = x^2 e^{3x}$.

OR

2 (a) Prove that $\dfrac{1}{D-a}x = e^{ax}\int e^{-ax} \cdot x dx$. **18**

(b) Discuss the continuity of $f(x, y) = \dfrac{x \cdot y^2}{x^5 + y^3}$ at the pt. $(0, 0)$

(c) Show that

$$e^{(ax+by)} = 1 + (ax+by) + \frac{(ax+by)^2}{2!} + \frac{(ax+by)^3}{3!} + \ldots\ldots$$

3 (a) In usual notations prove that **18**

$\dfrac{1}{f(D)}e^{ax} \cdot V = e^{ax}\dfrac{1}{f(D+a)}V$ where V is a function of x.

(b) Verify Eulers theorem for

$f(x, y) = \tan^{-1}\dfrac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}$; $(x, y) \neq (0, 0)$

(c) Solve $(x+a)^2\dfrac{d^2y}{dx^2} - 4(x+a)\dfrac{dy}{dx} + 6y = x$

OR

3 (a) Prove that $\frac{1}{(D-a)^r}e^{ax} = \frac{x^r \cdot e^{ax}}{r!}$; $\forall\ r \in N$ **18**

(b) Show that $e^{ax}\sin by = by + abxy + \frac{1}{6}\left(3a^2bx^2y - b^3y^3\right) + ...$

(3) Find the extreme value of

$f(x, y) = 2(x-y)^2 - x^4 - y^4$

4 (a) Prove that **18**

$$\beta(l, m) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{l+m}}dx = \int_0^\infty \frac{x^{l-1}}{(1+x)^{l+m}}dx = \beta(m, l)$$

(b) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$

then find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$

(c) Express $\int_0^1 x^m\left(1-x^n\right)^p dx$ in terms of Gamma function and

hence evaluate the integral $\int_0^1 x^5\left(1-x^3\right)^{10} dx$.

OR

4 (a) State and prove the relation between Beta and Gamma function. **18**

(b) Find the extreme value of $u = x^3y^2(1-x-y)$

(c) Show that $\int_0^\infty x^2e^{-x^4}dx \cdot \int_0^\infty e^{-x^4}dx = \frac{\pi}{8\sqrt{2}}$

5 (a) State and prove Taylor's theorem for bivariate function. **18**

(b) Evaluate $\int_0^4 \int_0^{\sqrt{16-x^2}} xy\ dxdy$

(c) Evaluate $\int_0^\pi \int_0^{\sin x} y\ dxdy$

OR

5 (a) If $\theta = x\frac{d}{dx}$ and $f(\theta)$ is a polynomial in θ with constant coefficients then prove that **18**

$$\frac{1}{f(\theta)}x^m = \frac{1}{f(m)}x^m; \text{ if } f(m) \neq 0$$

(b) If $f(x, y) = \sqrt{x^2 - xy}$ then prove that

$$x^2 f_{xx} + 2xy\, f_{xy} + y^2 f_{yy} = 0$$

(c) Obtain $\iint_S xy\, dx\, dy$ where S is bounded by $x = 2a$ and $y^2 = 4ax$.

6 (a) Solve by Gauss Elimination method **18**

$x + y + z = 9$, $2x - 3y + 4z = 13$, $3x + 4y + 5z = 40$

(b) Solve by Gauss Jordan Method

$2x + 3y + z = 9$, $x + 2y + 3z = 6$, $3x + y + 2z = 8$

(c) If $z(x + y) = x^2 + y^2$ then prove that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

OR

6 (a) Solve by Gauss Jordan Method **18**

$5x - 2y + z = 4$, $7x + y - 5z = 8$, $3x + 7y + 4z = 10$

(b) Solve by Seidel Method

$2x_1 + x_2 - x_3 = -1$, $x_1 - 2x_2 + 3x_3 = 9$, $3x_1 - x_2 + 5x_3 = 14$

(c) If $f(x, y) = \frac{x^2 + y^2}{x + y}$ then find f_{xx}, f_{yy}.